SPACETIME OR SPACE AND THE PROBLEM OF TIME

E. Anderson[1]

Astronomy Unit, Department of Mathematics, Queen Mary, London

In GR one may choose to use the spacetime action or its rearrangement[1][2]

$$\mathsf{I} = \int \mathrm{d}t \int \mathrm{d}^3x (p \circ \dot{h} - \alpha H - \beta^i H_i)$$

$$H \equiv \frac{1}{\sqrt{h}}\left(p_{ij}p^{ij} - \frac{p^2}{2}\right) - \sqrt{h}R = 0, \ , \ H_i \equiv -2D_j p_i{}^j = 0,$$

that corresponds to slicing spacetime into a sequence of 3-spaces. H and H_i are the Hamiltonian and momentum constraints. Their presence indicates redundancy in the (h_{ij}, p^{ij}) description of GR. H_i just indicates that the physically relevant information is not in whichever 3-space coordinate grid is used, but rather in the shape of the 3-space itself (its 3-geometry). Thus GR may be interpreted as a theory of evolving 3-geometries ('geometrodynamics'). H corresponds to GR spacetime being invariant under changes of slicing (corresponding to different choices of time w.r.t which this dynamics is understood to occur). Within any particular slicing this invariance is hidden, and we do not know if or how dynamics can be disentangled from the choice of time: one has a Problem of Time (POT) in geometrodynamics. This contributes much to rendering quantum geometrodynamics intractable.

So H is central to geometrodynamics. Wheeler asked[2] whether its form could be accounted for not from rearranging GR but rather from "plausible first principles". By allowing embeddability into spacetime to dictate the constraint algebra, the form of H was explained[3] using the algebra of (true and mere grid-stretching) deformations as plausible first principles.

However, recently 3-space – rather than spacetime – principles were proposed[4]. These develop Machian relationalism: physical laws whose validity exends to the universe as a whole should depend on relative quantites alone and contain no overall notion of time. In particle mechanics, these respectively involve distances and angles alone being meaningful, and the use of the time-label reparametrization-invariant (RI) Jacobi principle (which takes the form $\mathsf{I} = \int \mathrm{d}t\sqrt{\text{potential term} \times \text{kinetic term}}$ for the kinetic term homogeneous quadratic in its velocities). Moreover, an indirect choice ['best matching' (BM)] is made for the former: the action is not written in terms of relative quantities, but rather auxiliaries associated with absolute (rather than relative) motion are adjoined as corrections to the velocities. Then auxiliary-variation gives constraints which renders all absolute motion redundant. Now, it turns out that GR is indeed of this form. The absolute structure naïvely in the 3-space coordinate grids is redundant by the shift-corrections to the velocities which variationally-encode H_i. And upon algebraic elimination of the lapse, GR admits the RI BM action[5]:

$$\mathsf{I} = \int \mathrm{d}\lambda \int \mathrm{d}^3x \sqrt{h}\sqrt{RT}, \tag{1}$$

$$T = (h^{ik}h^{jl} - h^{ij}h^{kl})(\dot{h}_{ij} - \pounds_\xi h_{ij})(\dot{h}_{kl} - \pounds_\xi h_{kl}), \tag{2}$$

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[2] Here h_{ij} is the induced metric of a 3-space with determinant h, Ricci 3-scalar R and conjugate momentum p^{ij} with trace p. In moving between nearby 3-spaces, the lapse α is the time elapsed and the shift β_i is the ammount of stretching of the 3-space coordinate grid.

Thus in this sense GR is a perfectly Machian theory. Note now that H arises 'by Pythagoras' as a primary constraint.

The idea is to consider a large class of BM RI actions in place of (1). In each case H_i is guaranteed from the inbuilt, entirely spatial shift-variation, and a candidate H' arises 'by Pythagoras'. Now, applying Dirac's procedure[6] to H

$$(1 - X)sD_i(N^2 D^i p) = 0, \tag{3}$$

is required for consistency, where X represents the departure from –1 of the relative coefficients in the first factor of (2) and the potential $sR + \Lambda$ (const.) is in use (other potentials tried are all inconsistent). The $X = 1$ case is a derivation of relativity via *obtaining* embeddability into spacetime. But this arises in conjunction with two other cases which *do not* have spacetime structure. First, strong gravities[7] ($s = 0$), which generalize the strong-coupled limit of GR applicable near the Big Bang. Second, conformal theories[8] in which maximal ($p = 0$) or constant mean curvature (CMC) ($\frac{p}{h}$ = spatial constant) slicings are privileged; these include our *conformal gravity*[8]), and are closely tied to the GR initial value problem (IVP).

We next seek inclusion of fundamental matter to test the plausibility of our 3-space first principles. We then discover[4] [9] that via the Dirac procedure GR forces simple matter fields to share its null cone, and that electromagnetism and Yang–Mills theories arise. I play down claims about the unique selection of these theories in[10], but argue that Dirac theory and all the interaction terms of particle physics can be included[10]. This follows from Kuchař's spacetime split formalism[11] guaranteeing consistency. Although naïvely this would usually require spacetime *tilt* and *derivative-coupling* kinematics in addition to BM, I show that the entirely-spatial BM kinematics alone suffices to build an action for GR coupled to *all* of the above matter fields.

The 3-space – as opposed to spacetime – ontology, attempts to restore the centrality in dynamics of the configuration space of instants rather than the spacetime arena. This moves toward attempting a naïve Schrödinger interpretation (NSR) resolution of the POT. Alas, the geodesic principle idea from the analogy with Jacobi is of no aid[10]. Also, spacetime does not always emerge in the 3-space approach. It does not in the conformal theories, which have nevertheless a number of orthodox features such as locally-Lorentzian physics and IVP's similar to GR. I elect to explain here how these and the quantum-cosmologically-relevant strong gravity theories avoid some of GR's pitfalls in a number of POT strategies[12].

In GR, use of (h_{ij}, p^{ij}) variables and imposing H after quantization gives the Wheeler–DeWitt equation for which the Schrödinger i.p is indefinite and so not probabilistic. If one proceeds by analogy with Klein–Gordon, one is then floored by the indefinite sign of the general R-potential. But for conformal gravity, and a range of other conformal and strong gravities the Schrödinger i.p itself works, whereas for the other strong gravities the potential Λ is of definite sign.

Were one able to produce a mythical canonical transformation in GR to pass to separate embedding and dynamical variables (an approach with spacetime ontology connotations), by construction one would be equipped with a satisfactory i.p. In fact the York IVP method furbishes such a mythical transformation (corresponding to a CMC internal time), but this is unuseble because it involves the implicit solution of the Lichnerowicz PDE (conformalization of H) which is only tractable numerically. But for strong gravity theories the absence of R renders algebraic the corresponding equation, thus opening up this route.

Were I to show that any of our alternatives explain the very early universe or even all of nature, then the corresponding differences above would become directly important. In the absence of this, what I am doing is assessing which features of the GR POT obstructions are robust to changes of gravitational theory. These issues are work in progress.

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